Twenty-Third Meeting of the
Informal South Pacific ATC Co-ordinating Group (ISPACG/23)

Santiago, Chile, 26-27 March 2009

Agenda Item 4: Review Open Action Items

SUMMARY OF THE INVESTIGATION INTO THE USE OF AUTOMATIC DEPENDENT SURVEILLANCE-BROADCAST DATA FOR MONITORING AIRCRAFT ALTIMETRY SYSTEM ERROR

(Presented by US Federal Aviation Administration)

SUMMARY

This paper presents a summary of the investigation into the use of Automatic Dependent Surveillance – Broadcast for monitoring aircraft Altimetry System Error (ASE). The purpose of this paper is to provide details of the test flights conducted and the test results which support the use of ADS-B data to monitor the height-keeping performance of aircraft.

1. INTRODUCTION

1.1 The FAA William J. Hughes Technical Center presented a working paper to the Fourteenth Meeting of the Separation and Airspace Safety Panel Working Group of the Whole (SASP-WG/WHL/14) with details of initial test flights conducted to investigate the use of Automatic Dependent Surveillance – Broadcast (ADS-B) data for the estimation of aircraft Altimetry System Error (ASE). That working paper is attached to this paper as Attachment A.

1.2 The results of the initial test flights showed that the use of ADS-B for estimating aircraft ASE appeared promising. However, all of the initial test flights were completed using Global Positioning System (GPS) receivers equipped with Wide Area Augmentation System (WAAS) corrections. The FAA William J. Hughes Technical Center conducted additional test flights with the Wide Area Augmentation System (WAAS) corrections disabled. The purpose of these additional test flights was to determine whether ADS-B-derived aircraft geometric height data obtained from a GPS receiver with WAAS corrections or without WAAS corrections are sufficient for estimating aircraft ASE.

1.3 The purpose of this paper is to present details from the additional test flights and provide the results which support the use of ADS-B-derived aircraft geometric height data in estimating aircraft ASE.

2. DATA SOURCES

2.1 ADS-B Data

2.1.1. The ADS-B data obtained from the test flights were collected using two different systems. Both systems collect data from closely spaced GPS antenna mounted on the top of the aircraft. The data are then sent to two separate GPS receivers, the Universal Access Transceiver (UAT) system and the 1090 Extended Squitter (ES) system. The difference in the ADS-B derived aircraft geometric heights in the additional test flights are the WAAS corrections. The WAAS corrections are enabled on the GPS receiver for the UAT system.

The WAAS corrections were disabled on the GPS receiver for the 1090 ES system. Both of the GPS receivers, for the UAT and 1090 ES systems, met the criteria for Technical Standards Order (TSO) C145/146.

2.1.2. The treatment of the GPS-derived geometric height is different for each of the ADS-B systems involved – UAT and 1090 ES. The different treatment affects the accuracy and the variability of the resultant data received on the ground. The UAT system rounds the geometric altitude to the nearest 25 feet (ft) increment then a transmitter contained in the system sends the geometric altitude and other information to a ground receiver located at the FAA William J. Hughes Technical Center. The 1090 ES system collects the pressure altitude along with the difference between the geometric altitude and the pressure altitude. Prior to sending the data to the ground receiver each of these values is rounded to the nearest 25 ft increment. The 1090 ES system uses the aircraft's Mode S transponder to send the data to a ground receiver located at the FAA Technical Center.

2.1.3. The time field in both the 1090 ES messages and the UAT data is in Coordinated Universal Time (UTC). The geometric altitude is reported in feet. The aircraft geometric height data obtained from both ADS-B sources are quantized to 25 ft.

2.2 Truth Data

2.2.1. The geometric heights contained in the truth data are obtained from an independent Ashtech Inc. GPS receiver. The truth data are post processed using Novatel's software called GrafNav/GrafNet version 6.03. This post processing procedure improves the accuracy of the data by using information collected at various ground stations. The ground stations are arrayed in the local region and their positions are known, allowing corrections to be determined that are subsequently applied during post-processing. The time field in the truth data is in GPS time (14-second offset from UTC at the time of data collection, since 1 January 2009 GPS to UTC offset is 15 seconds (reference 1)), the geometric altitude is measured in meters with precision to the ten thousandth of a meter. This system is a recognized position reference.

3. METHODOLOGY

3.1 In order to determine if the geometric heights contained in the ADS-B data are suitable for the estimation of ASE, a comparison is made between the geometric heights from both sources of ADS-B data with the truth data. With the truth data considered absolute reference, this comparison is sufficient in determining if the ADS-B data can be used to estimate aircraft ASE because the geometric heights are a direct input to the process which will compute ASE values and will be treated in the same manner regardless of the source of the data.

3.2 The results from the initial test flights, described in Attachment A, also contained comparisons with aircraft geometric height data obtained from the Enhanced GPS Monitoring Unit (EGMU). The EGMU is one of several ways used world-wide to monitor the height-keeping performance of aircraft to determine its suitability for Reduced Vertical Separation Minima (RVSM) purposes. These initial results showed that, under carefully controlled conditions, the ADS-B derived aircraft geometric height data obtained from both ADS-B data sources (UAT and 1090 ES) are sufficient for estimating aircraft ASE. Since the WAAS enabled ADS-B source of aircraft geometric height was proven to be at least as good as the same obtained from the EGMU, a source that has been proven reliable in the estimation of aircraft, comparisons between the WAAS enabled and WAAS disabled ADS-B sources with the truth data were examined from the additional test flights.

3.3 WAAS corrections may not always be applied in a 1090 ES system, which may lead to differences in the geometric heights collected by the system. Three additional test flights were conducted by the FAA William J. Hughes Technical Center with the WAAS corrections disabled to determine whether the ADS-B aircraft geometric height obtained from all 1090 ES systems are suitable for estimating aircraft ASE. The additional analysis is critical because the operations conducted within RVSM flight levels (FL)290 through FL410, require periodic monitoring for ASE performance. If aircraft geometric height data obtained from ADS-B were to be used as a substitute in the RVSM monitoring processes, the data would very likely be in the 1090 ES format.

3.4 It is noted that all the test flights, including the WAAS disabled and WAAS enabled test flights, were conducted under stable conditions. The same airframe, flight path, and similar weather conditions were present for all test flights. In addition, the test flights took place in the mid-latitude region of the northern hemisphere during the solar minimum, or the start of solar cycle 24 (References 2 and 3). The solar minimum is the lowest point of the sun's 11-year average activity cycle (References 2 and 3). During this time, the rate of solar storms, solar flares, and sun spots is expected to be low. Therefore, any disturbance to the onboard GPS system caused by solar activity is expected to be minimal.

3.5 The additional test flights were conducted in the same manner as those described in Attachment A. Aircraft geometric height data were collected from a total of 10 level flight segments from the test flights. The results of the data are described in the next section.

4. DISCUSSION

4.1 The results from the three additional test flights, with the WAAS corrections disabled are shown in Figure 1. The box plots in Figure 1 show the ranges of the data sets. The horizontal bars above and below the boxes represent the highest and lowest value of the data sets respectively. The bottom and top of each red box is the lower and upper quartile, respectively. The distance between the lower and upper quartile, the inter-quartile range, provides a measure of the spread of the distribution. The white line in the center of the red box is the median of the data. Quartiles are defined as the point where the data is divided into four equal parts, meaning that there are an equal number of data points between each quartile. Figure 1 shows the data range observed from the two sources are different.

4.2 These additional test flights produced similar data to that collected from the first set of test flights, shown in Attachment A. The geometric height data range observed from the UAT system is much smaller than that of the 1090 ES system. The geometric height data range observed from the UAT system and the 1090 ES system without WAAS corrections was 5.238 ft and 16.604 ft, respectively. The performances of the systems used to monitor aircraft height-keeping performance have an overall known error of 35 ft. The items that contribute to the overall known error include internal elements, specific to the monitoring system itself, and external elements, such as the weather data.

4.3 Analysis of Variance (ANOVA) is a statistical test used to test for differences among two or more independent samples. In this study the independent samples are each of the measurement systems. In this case, comparisons will be made between the differences between each of the sources and the truth data. Each level flight segment of the test flights represents one replication. The null hypothesis for this test is that the mean differences in geometric heights from both the WAAS disabled and WAAS enabled systems when compared to the truth data are equal. The null hypothesis is tested at a 95% confidence level. The ANOVA test is shown in Table 1.

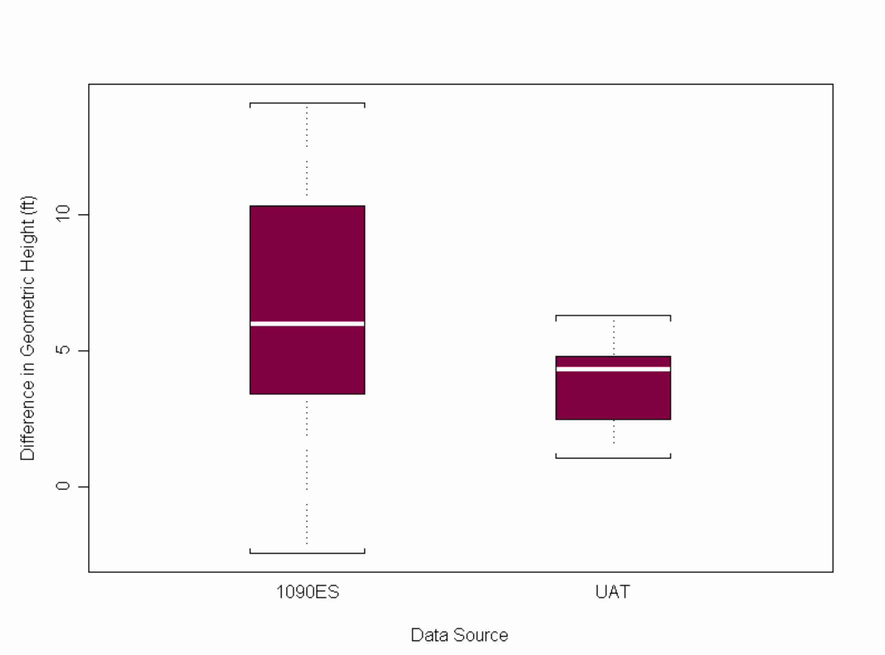

Figure 1. Box Plots of the 1090 ES and UAT Aircraft Geometric Height Differences from Truth Data for Test Flights with WAAS Corrections Disabled (1090ES) and Enabled (UAT)

Summary				
Groups	**Count**	**Sum**	**Average**	**Variance**
1090 ES Without WAAS	10	60.578	6.0578	24.07581
UAT With WAAS	10	37.63059	3.763059	2.882348

ANOVA

Source of Variation	**SS**	**df**	**MS**	**F**	**P-value**	**F crit**
Between Groups	26.32919	1	26.32919	1.9533	0.1792	4.414
Within Groups	242.6234	18	13.47908			
Total	268.9526	19				

Table 1. Analysis of Variance Test for the Difference between Truth Data and the 1090 ES Without WAAS and UAT With WAAS Data

4.4 The results in Table 1 show that at a 95 percent confidence level the null hypothesis cannot be rejected. The results of this test indicate the means of the 1090 ES without WAAS corrections and the UAT with WAAS corrections are not significantly different. All of the data comparisons shown in Table 1 came from the same test flights. It was not possible to collect both WAAS disabled and WAAS enabled data from the 1090 ES system. However, the initial test flights presented in Attachment A determined that with one fewer rounding error than the 1090 ES system, the UAT system produces slightly more accurate results. Therefore, the successful comparisons with the UAT system are enough to demonstrate that the WAAS disabled, or aircraft geometric height data obtained from a system without WAAS corrections is sufficient for estimating aircraft ASE.

4.5 It is possible that the stable atmospheric conditions described in paragraph 3.4 influenced the results of the WAAS disabled test flights. The test flights were conducted in the mid-latitude region during a period of low solar activity which minimized any atmospheric disturbance on the GPS systems. Due to the potential erroneous outcomes from GPS systems which can result under less stable conditions, it is recommended that any height-monitoring system utilizing aircraft geometric height data obtained from a GPS system without WAAS corrections include additional quality control procedures specifically to evaluate the existing atmospheric conditions during the monitoring period.

4.6 These additional quality control procedures needed for GPS aircraft geometric height data supplement the basic quality control procedures of any ground-based monitoring system. The purpose of these quality control procedures are to help prevent the identification of an airframe as having good height-keeping performance when in fact the true height-keeping performance is not acceptable (Type II error).

5. ACTION BY THE MEETING

5.1 The meeting is invited to:

a) Note the information provided in this paper

b) Endorse the continued exploration of ADS-B derived geometric height as a data source for aircraft height-keeping performance monitoring.

c) Consider whether aircraft height-keeping performance monitoring can be conducted using ADS-B in the South Pacific region.

REFERENCES

1. United States Naval Observatory, “GPS Timing Data & Information”, http://tycho.usno.navy.mil/gps_datafiles.html.
2. National Oceanic and Atmospheric Administration (NOAA) Space Weather Prediction Center, “Space Weather Services for GPS”, September 2007, http://www.navcen.uscg.gov/cgsic/meetings/47thMeeting/%5B10%5D%20Kunches%20Ft.%20Worth%20Sept.%2007.pdf.
3. National Oceanic and Atmospheric Administration (NOAA) Space Weather Prediction Center, “Solar Cycle Progression”, 31 January 2009, http://www.swpc.noaa.gov/SolarCycle/.

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REV. 01

International Civil Aviation Organization

WORKING PAPER

SEPARATION AND AIRSPACE SAFETY PANEL (SASP) MEETING OF THE WORKING GROUP OF THE WHOLE

FOURTEENTH MEETING

Paris, France, 13 to 24 October 2008

An Update to the Investigation into the Use of Automatic Dependent Surveillance-Broadcast Data for Monitoring Aircraft Altimetry System Error

(Presented by Madison Walton)
(Prepared by Lauren Martin and Christine Falk)

SUMMARY

This paper presents an update to the work the Federal Aviation Administration (FAA) has undertaken to compare aircraft geometric height data obtained from Automatic Dependent Surveillance – Broadcast (ADS-B) messages with differentially corrected GPS geometric height data. This paper provides details of flight tests conducted by the FAA Technical Center. This paper also provides the test results which support the use of ADS-B data to monitor the height-keeping performance of aircraft.

1. Introduction

1.1 Anticipating the adoption of global of long-term monitoring requirements, the Eighteenth Meeting of the Asia/Pacific Air Navigation Planning and Implementation Regional Group (APANPIRG) considered the consequences of such requirements within the Asia-Pacific Region (reference 1, paragraph 5.10). That Meeting also considered the role of two new technologies – Automatic Dependent Surveillance – Broadcast (ADS-B) and multilateration – within the Region and adopted several important decisions regarding them.

1.2 ADS-B samples data taken from a GPS-derived position of the aircraft and sends that position along with other aircraft-dependent data via datalink to ground receivers that forward the data to a common site. Part of the data sent is the estimate of the geometric height of the aircraft. It was proposed (reference 2) that these estimates of geometric height may be suitable for use in estimating aircraft altimetry system error (ASE).

1.3 Initial comparisons of aircraft geometric height data obtained from ADS-B sources were provided in reference 3. Reference 3 also provided a plan for test flights to be conducted by the FAA Technical Center to help determine whether aircraft geometric height obtained from ADS-B sources are sufficient for estimating aircraft ASE. A complete description of the test flights and some of the initial results were published in reference 5. This paper provides an updated summary of the material contained in reference 5. This paper also includes results not included in reference 5 obtained from recent test flights.

2 Background

2.1 ASE is a measure of the height-keeping performance of an aircraft. In airspace where the Reduced Vertical Separation Minimum (RVSM) is applied, the importance of accurate aircraft height-keeping is magnified. ASE is not detectible in routine operations; specialized measurement equipment is necessary to independently measure the errors. If an aircraft is unable to maintain its desired altitude relative to others, it poses a greater threat to the other aircraft in the system.

2.2 In preparation for the implementation of the RVSM, the FAA developed a process to determine Total Vertical Error (TVE), ASE and Assigned Altitude Deviation (AAD). Figure 1 provides an illustration of the relationship between these errors. One method of estimation uses a portable device, called the Enhanced GPS-based monitoring unit (EGMU) which is placed on board an aircraft; it collects GPS pseudo-ranges through the aft windows on the flight deck. These data are then differentially corrected to improve their accuracy and aircraft position is estimated, which results in aircraft geometric height data. The corrected geometric height information is compared to the geometric height of the flight level flown by the aircraft, with the latter obtained using global meteorological model data. The EGMU also collects Mode C returns for the flight with its Altitude Recorder Device (ARD) component, producing data used to estimate AAD. All three of these data sources are then combined in a process which estimates TVE and ASE.

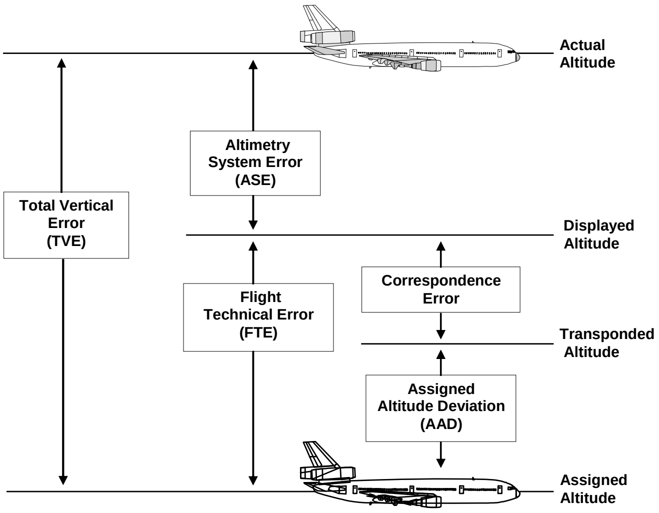

Figure 1. Components of Total Vertical Error (TVE)

2.3 Aircraft ASE is computed by EUROCONTROL and the North Atlantic (NAT) Central Monitoring Agency (CMA) using a ground-based system, the Height Monitoring Unit (HMU). There are four HMUs - three in Europe and one in the United Kingdom. The FAA also uses a ground-based system, the Aircraft Geometric Height Measurement Element (AGHME). Unlike the HMU which produces estimates of TVE, ASE and AAD directly, the AGHME estimates only aircraft geometric height. The FAA's process for determining the estimates of TVE and ASE using AGHME-derived aircraft geometric height is the same way as that described in paragraph 2.2. Currently, there are five AGHME systems operational in North America, 3 in the United States and 2 in Canada.

2.4 An ADS-B-equipped aircraft uses an on-board GPS receiver to determine its position. This time-stamped information is then broadcast along with other information to all ADS-B capable aircraft and to ADS-B ground or satellite communications stations. These stations then forward the information to air traffic control centers. The ADS-B message includes aircraft geometric height and pressure altitude in reference to a standard atmosphere, which are key components in the ASE estimation process.

2.5 The geometric height obtained from the EGMU is differentially corrected prior to the ASE calculation. This means that the position errors are removed from the GPS-derived geometric height with further processing. During development of the EGMU, the FAA Technical Center determined that the uncorrected aircraft geometric height produced using EGMU-collected pseudo-ranges was not of sufficient accuracy to support adequate estimation of TVE, AAD and ASE.

2.6 The GPS-derived geometric height contained in the ADS-B message is not differentially corrected. It is not possible to post-process these geometric heights because the information needed to correct the errors is not included in the ADS-B messages. Some conditions have changed since the initial determination of suitability of uncorrected GPS pseudo-ranges. First, aircraft grade GPS receivers have improved markedly and being capable of tracking more satellites simultaneously. Additionally, the Selective Availability (SA) feature of the GPS system has been completely disabled to the point where non-precision approaches can be attempted with its course guidance. These changes in conditions mean that better accuracy can be expected in the geometric height determined from the modern receivers. Therefore, the underlying issue is whether the uncorrected GPS geometric height contained in the ADS-B message is sufficiently accurate to support aircraft height monitoring.

3 Discussion

3.1 Test Flight Details

3.1.1 This section contains a summary of the test flight description information provided in reference 5.

3.1.2 The FAA Technical Center has a fleet of research aircraft that are used for conducting tests and evaluations of avionics systems. One of these aircraft, N47 – a Bombardier BD-700-1A11 aircraft, was used for this study. This aircraft has ADS-B capabilities, but prior to our study, was not used for ADS-B test purposes at altitudes above FL290. Therefore, data collection equipment needed to be adjusted to allow for the receipt of the ADS-B messages to a ground receiver prior to conducting the test flights. In addition a 1090 Extended Squitter (ES) suite of test avionics had to be constructed for the aircraft. The aircraft is equipped with two GPS antenna on top of the aircraft. Each antenna provides a source for independent GPS data, the study refers to these data as truth data. The antenna located on the top right side of the fuselage is the source for one set of truth data along with the Universal Access Transceiver (UAT) Data Link. The antenna located on the top left side of the fuselage is the source for a second set of truth data and the 1090 ES.

3.1.3 There were three initial test flights conducted for this study. Each test flight consisted of a series of four level flight segments. The flights departed from the Atlantic City International Airport (ACY) in Atlantic City, New Jersey. An illustration of the test flight path is shown in Figure 2.

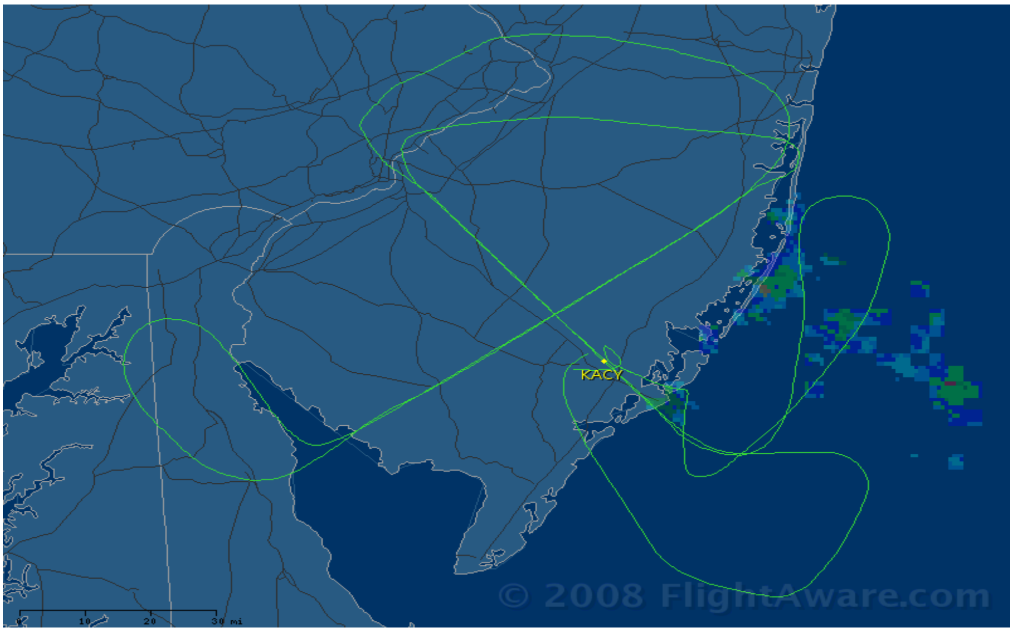

Figure 2. Sample Test Flight Path

3.1.4 All four segments of the first test flight were flown at FL280. The first two segments of the second test flight were flown at FL280 and the second two segments were flown at FL410. All data types, truth data, EGMU, UAT and 1090 ES, were collected for the first two test flights. The first two segments of the third test flight were flown at FL280 and the last two segments were flown at FL410. Due to a problem with the 1090 ES receiver, it was not possible to collect 1090 ES data during the third test flight; UAT, EGMU and truth data were collected. Table 1 summarizes the test flights, including the length of each segment and the altitude.

Date of Test Flight and Segment Number	**Duration (minutes)**	**FL**
June 26 – Segment 1	9	280
June 26 – Segment 2	12	280
June 26 – Segment 3	10	280
June 26 – Segment 4	7	280
June 27 – Segment 1	10	280
June 27 – Segment 2	11	280
June 27 – Segment 3	10	410
June 27 – Segment 4	8	410
July 2 – Segment 1	8	280
July 2 – Segment 2	12	280
July 2 – Segment 3	3	410
July 2 – Segment 4	7	410

Table 1. Test Flight Details

3.2 Description of Data Obtained From Test Flights

3.2.1 This section contains a summary of the data description provided in reference 5.

3.2.2 ADS-B Data

3.2.2.1. The ADS-B data obtained from the test flights were collected using two different systems. Both systems collect data from closely mounted GPS antenna mounted on the roof of the aircraft. The data is then sent to two separate Wide Area Augmentation System (WAAS) enabled GPS receivers, the UAT system uses a Garmin GPS receiver and the 1090 ES system uses a Rockwell Collins GNLU-930 Multi-Mode Receiver. The UAT system and Garmin receiver are typical of general aviation type aircraft while the 1090 ES system and Rockwell Collins receiver are typical of commercial type aircraft.

3.2.2.2. The treatment of the GPS-derived geometric height is different for each of the ADS-B systems involved – UAT and 1090 ES. The different treatment affects the accuracy and the variability of the resultant data received on the ground. The UAT system rounds the geometric altitude to the nearest 25 ft increment then a transmitter contained in the system sends the geometric altitude and other information to a ground receiver located at the FAA Technical Center. The 1090 ES system collects the pressure altitude along with the difference between the geometric altitude and the pressure altitude. Prior to sending the data to the ground receiver each of these values is rounded to the nearest 25 ft increment. The 1090 ES system uses the aircraft's Mode S transponder to send the data to a ground receiver located at the FAA Technical Center.

3.2.2.3. The time field in both the 1090 ES messages and the UAT data is in Coordinated Universal Time (UTC). The geometric altitude is reported in feet. The aircraft geometric height data obtained from both ADS-B sources are quantized to 25 ft.

3.2.3 Truth Data

3.2.3.1. The geometric heights contained in the truth data are obtained from an independent Ashtech Inc. GPS receiver. The truth data are post processed using Novatel's software called GrafNav/GrafNet version 6.03. This post processing procedure improves the accuracy of the data by using information collected at various ground stations. The time field in the truth data is in GPS time (currently a 14-second offset from UTC), the geometric altitude is measured in meters with precision to the ten thousandth of a meter.

3.2.4 EGMU Data

3.2.4.1. Additionally, an EGMU was brought onboard the aircraft. Data from this system were processed in the same manner established for monitoring ASE in the initial implementation of the RVSM. The aircraft geometric height data obtained from the EGMU is differentially corrected through post-processing. Meteorological data, needed to determine the geometric height of the assigned flight level, are obtained from the National Weather Service. The time in the EGMU data is collected using GPS time and the geometric altitude is measured in feet with precision to the one hundredth of a foot.

3.3 Methodology

3.3.1 In order to determine if the geometric heights contained in the ADS-B data would be suitable for the estimation of ASE a comparison is made between the geometric heights in both sources of the ADS-B data and the EGMU data with the truth data. This comparison is sufficient in determining if the ADS-B data can be used to estimate aircraft ASE because the geometric heights are a direct input to the process which will compute ASE values and will be treated in the same manner regardless of the source of the data. Currently, many aircraft ASE estimates in the United States are computed using data collected by the EGMU. It is important to compare the results obtained from the EGMU with the results obtained from ADS-B data. These comparisons are necessary to determine whether the ADS-B geometric heights are as good as those obtained from a source that has been proven reliable in the estimation of ASE.

3.3.2 The time information contained in the truth data is GPS time. The time information in the ADS-B data are in UTC time. Currently, GPS time is ahead of UTC by 14 seconds. Corrections were made to the time information prior to matching the data for the comparisons.

3.3.3 A correction was applied to the EGMU data to account for the vertical displacement between the locations of the antenna which measure the geometric height of the aircraft. The EGMU uses an antenna that is mounted on the window of the aircraft. The geometric altitude in the truth data is determined from an antenna mounted on the top of the aircraft. The distance between the center of the window and the top of the aircraft was 41.5 inches. This amount was subtracted from the difference between the geometric height in the truth data and the geometric height in the EGMU data.

3.3.4 Once the data were time matched, and the level flight segments were identified, the geometric height difference between the truth data and the other available sources (EGMU, UAT and 1090 ES) was computed. The average difference from each source is displayed in Figure 3.

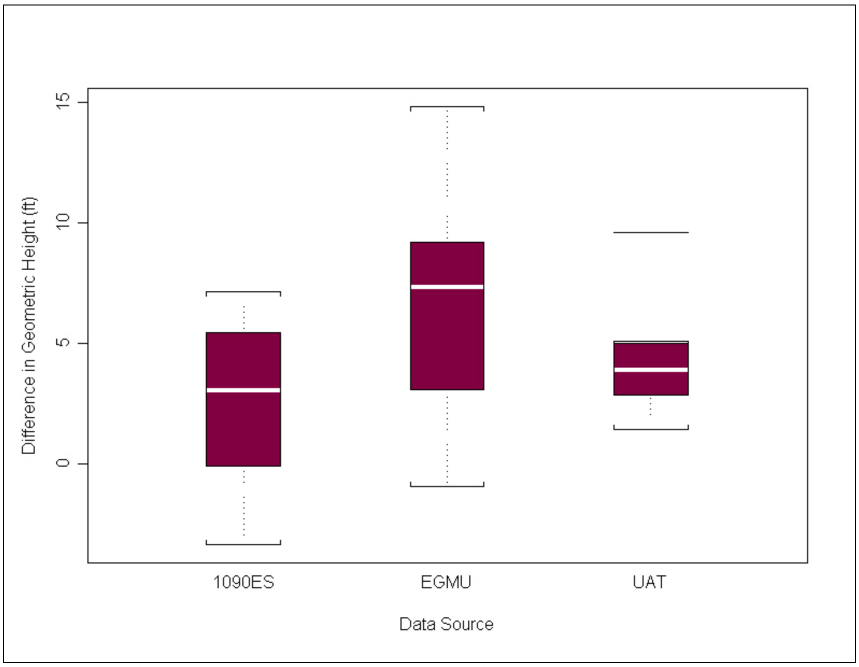

Figure 3. Box Plots of the Aircraft Geometric Height Difference between the Truth Data and the Available Sources (1090 ES, EGMU, and UAT)

3.3.5 The box plots in Figure 3 show the ranges of the data sets. The horizontal bars above and below the boxes represent the highest and lowest value of the data sets respectively. The bottom and top of each red box is the lower and upper quartile, respectively. The distance between the lower and upper quartile, the inter-quartile range, provides a measure of the spread of the distribution. The white line in the center of the red box is the median of the data. Quartiles are defined as the point where the data is divided into four equal parts, meaning that there are an equal number of data points between each quartile. Figure 3 shows the data range observed from the three sources are different. The largest range of data is observed for the EGMU data.

3.3.6 Figure 4 shows a comparison of the geometric heights from the 1090 ES, the UAT and the truth data. The time, in seconds, of the level flight segment is along the x-axis and the geometric altitude is on the y-axis. The UAT data and the 1090 ES data are plotted along with the truth data in order to show the differences in each of the data sets. The data are from the first level flight segment of the test flight on June 26, 2008.

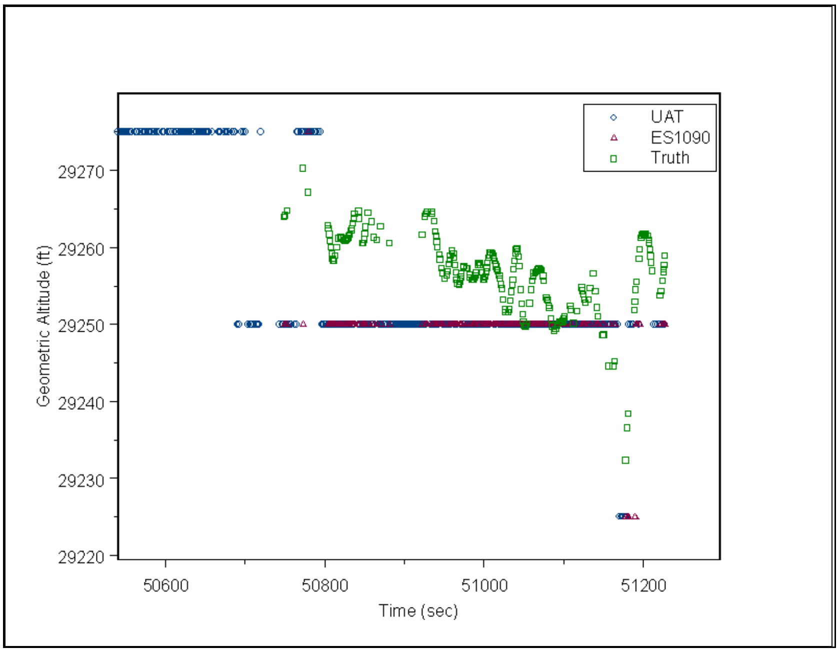

Figure 4. ADS-B and Truth data for June 26, 2008 Segment 1

3.3.7 Both the 1090 ES and UAT are quantized to 25 ft, while the truth data is accurate to one thousandth of a foot. Prior to conducting the test flights, it was expected that the aircraft geometric height from the two ADS-B sources, UAT and 1090 ES, would produce similar results when compared to the truth data. Both sources of ADS-B data obtain the aircraft geometric height information from identical antenna, in the same location on the aircraft and are both WAAS corrected.

3.3.8 The geometric heights collected by the UAT on board the aircraft are rounded to a 25 ft increment prior to being sent to the ground receiver. The expected value and the variance of the difference of the true aircraft geometric heights from the UAT data are defined in Appendix A. There are three potential outcomes for the expected value and variance of the UAT geometric height, which are determined by the rounding method. All three potential outcomes involve the addition of a rounding error to the geometric height data. In the first potential outcome, the geometric height data are rounded up or down to the nearest 25 ft increment. In this case, the rounding error is a uniform random variable with a range of -12.5 to 12.5 ft and a mean equal to zero. In the second potential outcome, the geometric height data are rounded up to the nearest 25 ft increment. In this case, the rounding error is modeled as a uniform random variable with a range of 0 to 25 ft and a mean equal to 12.5 ft. Finally, for the third potential outcome, the geometric height data are rounded down to the nearest 25 ft increment. In this case, the rounding error is a uniform random variable with a range of -25 to 0 ft and a mean equal to -12.5 ft. The UAT data collected for this study have an average difference from the truth data of 4.303 ft. This result supports the assumption that the rounding process for the UAT data follows the first potential outcome and is rounded up or down to the nearest 25 ft increment.

3.3.9 The 1090 ES collects the pressure altitude, which is rounded to a 25 ft increment. The 1090 ES also provides the difference between the aircraft geometric altitude and the pressure altitude, this difference is also rounded to a 25 ft increment. The pressure altitude and the difference between the aircraft geometric height and the pressure altitude are sent to the ADS-B ground receiver. During post processing, the difference between the aircraft geometric altitude and the pressure altitude are added to the pressure altitude to determine the aircraft geometric altitude. This addition produces an estimate of aircraft geometric altitude from the sum of two values which were previously rounded to a 25 ft increment. The expected value and variance of the true aircraft geometric height in relation to the 1090 ES data are presented in Appendix B.

3.3.10 Similar to the UAT geometric height data, there are also three potential outcomes for the expected value of the geometric height contained in the 1090 ES data as long as both sources receive similar treatment. Since the geometric height is determine by adding two values that are rounded, the rounding error added to the geometric height in the 1090 ES data is a sum of two errors created during the rounding of both values. In the first potential outcome, the geometric height data are rounded up or down to the nearest 25 ft increment. In this case, each rounding error is a uniform random variable with a range of -12.5 to 12.5 ft and the mean of each error equal to zero. In the second potential outcome, the geometric height data are rounded up to the nearest 25 ft increment. In this case, the rounding errors are uniform random variables with ranges of 0 to 25 ft and the mean of each error is 12.5 ft. This leads to an overall error included in the expected value of the 1090 ES geometric height data of 25 ft. In the third potential outcome, the geometric height data are rounded down to the nearest 25 ft increment. In this case, the rounding errors are uniform random variables with ranges of -25 ft to 0 and the mean of each error is -12.5 ft. This leads to an overall error included in the expected value of the 1090 ES geometric height data of -25 ft. The data collected for this study have an average difference from the truth data of 2.581 ft, this means the 1090 ES geometric height data is on average 2.581 ft lower than the truth data. These data support the assumption that the rounding process for each element of the 1090 ES data follows the first potential outcome and are rounded up or down to the nearest 25 ft increment.

3.4 Results

3.4.1 Analysis of Variance (ANOVA) test are used to test for differences among two or more independent samples. In this study the independent samples are each of the measurement systems. In this case comparisons will be made between the differences between each of the sources and the truth data. Each level flight segment of the test flights represents one replication. Only the tests flights from June 26 and 27 will be used for this analysis because all four data sources were not available during the July 2 test flight. The null hypothesis for this study tests whether there is a difference between the mean differences in geometric heights when comparing each data source to the truth data. The null hypothesis is tested at a 95% confidence level. Table 2 contains the results of the ANOVA analysis.

Groups	**Count**	**Sum (ft)**	**Average (ft)**	**Variance (ft)**		
EGMU	8	52.987	6.623	25.812		
1090 ES	8	20.641	2.580	13.272		
UAT	8	34.428	4.303	5.979		
ANOVA						
Source of Variation	**SS**	**df**	**MS**	**F**	**P-value**	**F crit**
Between Groups	65.866	2	32.933	2.192	0.137	3.467
Within Groups	315.457	21	15.022			
Total	381.321	23				

Table 2. Analysis of Variance Test for the Difference in Average Geometric Height Values Obtained from Available Sources and Truth Data

3.4.2 The results in Table 2 show that at a 95 percent confidence level the null hypothesis can not be rejected. The results of this test indicate the means of the three samples are not significantly different.

3.4.3 The results in Table 2 also show that the EGMU data has the largest average difference from the truth data; it also has the largest variance. One reason for this result is the location of the antennae for the EGMU. During each flight, two EGMU antennae are positioned on the inside windows of the aircraft, one on a left side and right side window of the aircraft. Due to the limited view of the available satellites at these positions, the EGMU data can be "noisy". This "noise" is shown in the Table 2 results in the estimate of the EGMU variance.

3.4.4 Both the UAT and 1090 ES data used in this study were WAAS corrected. The FAA's ADS-B surveillance team indicated that all UAT systems use WAAS corrected data. Therefore, it is expected that the aircraft geometric heights obtained from other UAT equipped aircraft would have similar means and standard deviations if compared to truth data as shown in the results from this study. The EGMU is a proven and validated system for estimating aircraft ASE. Therefore, it is expected that ADS-B aircraft geometric height data obtained from a UAT system would produce similar ASE results as the EGMU. Because WAAS corrections are applied in all UAT systems, further testing for UAT aircraft geometric height data without WAAS corrections is not possible.

3.4.5 WAAS corrections may not always be applied in a 1090 ES system, which may lead to differences in the geometric heights collected by the system. Additional test flights were conducted by the FAA Technical Center with the WAAS corrections disabled to determine whether the ADS-B aircraft geometric height obtained from all 1090 ES systems are suitable for estimating aircraft ASE. The additional analysis is critical because the operations conducted within RVSM flight levels, FL290 through FL410, require periodic monitoring for ASE performance. If aircraft geometric height data obtained from ADS-B were used for the RVSM operations, the data would be in the 1090 ES format. Initial results of these test flights, with the WAAS corrections disabled are shown in Figure 5.

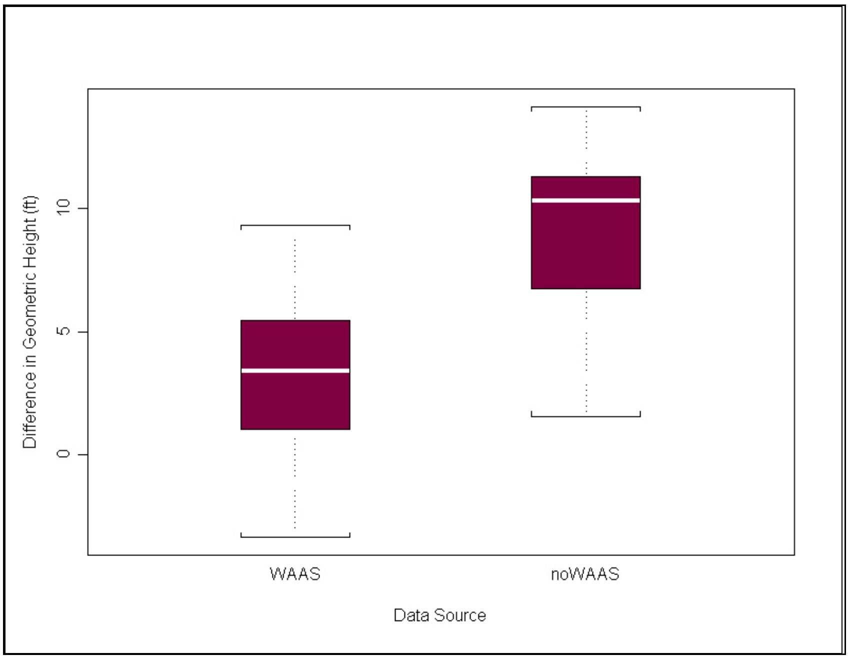

Figure 5. Box Plots of the 1090 ES Aircraft Geometric Height Difference from the Truth Data for Test Flights with WAAS Corrections Disabled (noWAAS) and Enabled (WAAS)

3.4.6 Additional test flights are planned to verify these results. It is not possible to collect both "without WAAS" and "with WAAS" 1090 ES data from the same test flight. Therefore, more samples are needed to increase the confidence in the results. The UAT data were available from the same "without WAAS" flights. Table 3 shows the geometric heights obtained from the "without WAAS" test flights.

	UAT - With WAAS Corrections		**1090 ES – Without WAAS Corrections**	
Date of Test Flight and Segment Number	**Average Difference in Geometric Height (feet)**	**Standard Deviation of Differences in Geometric Height**	**Average Difference in Geometric Height (feet)**	**Standard Deviation of Differences in Geometric Height**
Aug 7 – Segment 1	4.429	6.863	1.560	8.625
Aug 7 – Segment 2	2.486	7.187	6.720	6.579
Sep 30 – Segment 1	4.402	6.571	11.276	7.826
Sep 30 – Segment 2	1.142	6.529	14.134	9.584
Sep 30 – Segment 3	4.803	7.310	10.319	9.758
Sep 30 – Segment 4	4.681	7.503	Insufficient Data	Insufficient Data

Table 3. UAT (With WAAS) and 1090 ES (Without WAAS) Geometric Height Data Results

3.4.7 It is not expected that the UAT system with WAAS and the 1090 ES system without WAAS would have the same results. But, it is believed that more than five level flight segments are needed to make a final determination on the "without WAAS" data.

3.5 ASE Estimates

3.5.1 Table 4 presents the aircraft ASE estimates for the test flights computed using data collected from both the EGMU system and the ADS-B systems on June 27, 2008 and July 2, 2008. It was not possible to compute the ASE values for the test flight on June 26 because the ASE software, developed at the FAA Technical Center, utilizes data obtained from flights operating with RVSM flight levels, FL290 through FL410. The flight on June 26 was flown at FL280.

3.5.2 All of the aircraft ASE estimates shown in Table 4 were made using the aircraft ASE estimation process developed by the FAA, described in paragraph 2.2 and Figure 1. Aside from the aircraft geometric height information, all of the necessary inputs for the ASE estimation process, including the meteorological data and Mode C returns (or Mode S), were identical. All of the aircraft ASE estimates shown in Table 4 compare well. The aircraft ASE estimated with the aircraft geometric height obtained from the 1090 ES system was computed for June 27 only due to a problem with the ground receiver during the July 2 test flight. The ASE estimates from the "without WAAS" test flights are not yet available.

Segment and Flight Level	**Data Source**	**ASE**	**Number of Observations**
June 27 FL 410	EGMU	73	188
	1090 ES	70	176
	UAT	73	310
June 27 FL 410	EGMU	111	137
	1090 ES	124	65
	UAT	118	173
June 27 FL 410	EGMU	78	155
	1090 ES	Insufficient Data	5
	UAT	76	155
July 2 FL 410	EGMU	57	647
	UAT	58	58

July 2 FL 410	EGMU	55	468
	UAT	50	46

Table 4. Estimates of Aircraft ASE from the Test Flights

4 Conclusion

4.1 The results shown in this paper indicate that ADS-B data obtained from a UAT source is sufficient for estimating aircraft ASE. The results also show that the ADS-B data obtained from a 1090 ES source may also be a sufficient source for estimating aircraft ASE, pending more test flights conducted with the WAAS corrections disabled. It is important to note that the ASE estimation process considered in this study is the same process developed by the FAA for estimating ASE using the EGMU. The results from this study considered the difference in the geometric height of the aircraft from the various sources. The remaining necessary inputs for the estimate of aircraft ASE, such as the meteorological information needed to obtain the geometric height of the flight level, are the same regardless of the source of aircraft geometric height.

4.2 The initial results presented in this paper, pending additional test flights results conducted with WAAS corrections disabled, show that ADS-B is a suitable ASE monitoring solution. The use of ADS-B to estimate aircraft ASE would be one of several available monitoring options to operators and Regional Monitoring Agencies (RMAs) to satisfy monitoring requirements.

4.3 A RMA may use ADS-B data to estimate aircraft ASE for those aircraft utilizing ADS-B. Aircraft not utilizing ADS-B could not be monitored from these data. Therefore, a complete monitoring program, from the standpoint of a RMA, includes a combination of ground-based monitoring systems such as the HMU or AGHME, airborne systems such as the EGMU, and ADS-B. Ground-based monitoring systems give large volumes of data which provides information about the bulk of the airspace population. It also allows for repeated measurements on airframes which are significant in detecting trends in ASE performance. Airborne systems provide the ability to target specific portions of the airspace population to meet immediate needs. Since ADS-B systems are still being developed in many regions of the world, the use of ADS-B for monitoring holds promise, but conventional ground-based monitoring needs to fill the gap or be the permanent solution for the near-term.

5 Recommendation

5.1 The meeting is invited to note the information provided in this paper.

5.2 The meeting is invited to endorse the continued exploration of ADS-B derived geometric height as a data source for aircraft height-keeping performance monitoring.

5.3 The meeting is invited to consider, subject to the outcome of additional ADS-B test flights, if ADS-B derived aircraft geometric height prove to be suitable, whether aircraft height-keeping performance monitoring can be conducted using ADS-B in various regions.

References

1 "Report Of The Eighth Meeting Of The Regional Airspace Safety Monitoring Advisory Group (RASMAG/8)," Bangkok, Thailand, 10 – 14 December 2007.

2 "Use of ADS-B for Height-Keeping Monitoring," SASP WG/WHL/12-WP/22, Santiago, Chile, 5 - 16 November 2007.

3 “Investigation into the Use of Automatic Dependent Surveillance-Broadcast Data for Monitoring Aircraft Altimetry System Error”, SASP WG/WHL/13-IP/13, Montreal, Canada, 12-23 May 2008.

4 “Summary of the Comparison Between Demonstrator HMU and GMS”, ICAO North Atlantic (NAT) Systems Planning Group (SPG) Airspace Monitoring Subgroup (AMSG) Eighth Meeting, Shannon, Ireland, 23-27 June 1997.

5 Martin, L., Falk, C., Perez, J., “Investigation into the use of Automatic Dependent Surveillance-Broadcast Data for Monitoring Aircraft Altimetry System Error”, American Institute of Aeronautics and Astronautics (AIAA) Guidance, Navigation, and Control Conference, Honolulu, Hawaii, USA, August 18-21, 2008 (AIAA 2007-7146 - Invited Session).

Appendix A
UAT Aircraft Geometric Height Modeling

UAT aircraft geometric height estimates are GPS height measurements rounded in 25 foot increments and are derived from the aircraft's L1 WAAS enabled GPS receiver. The UAT height estimate is similar in nature to the 1090 geometric height estimate since both are derived from GPS data. However, the UAT height estimate has three distinct advantages over the 1090 data: (1) transmission of data is always from a WAAS enabled receiver; (2) rounding occurs once, as opposed to twice in the 1090 data; and (3) the messages do not have to be synchronized in time.

There are three models considered for the expected value and variance of the UAT geometric height. The three models are; 1) rounding up or down to the nearest 25 ft increment, 2) rounding up to the nearest 25 ft increment and, 3) rounding down to the nearest 25 ft increment.

Let h_u = rounded (GPS height) from the UAT geometric height estimate, then

h_u = rounded $(h_t + e_n)$.

Where h_t = true height of the aircraft, and e_n = random error $N(0,\sigma)$

Therefore h_u can be rewritten as $h_u = h_t + e_n + e_r$, where

e_r = rounding error $U[a,b]$, with $E(h_u)$ as

$$E(h_u) = E(h_t + e_n + e_r)$$

$$= E(h_t) + E(e_n) + E(e_r)$$

1. If the data is rounded up or down, then e_r is $U[a,b] = U[-12.5,12.5]$

$$E(h_u) = E(h_t) + E(e_n) + E(e_r)$$
$$= E(h_t) + 0 + 0$$
$$= E(h_t)$$

2. If the data are rounded up, then e_r is $U[a,b] = U[0,25]$

$$E(h_u) = E(h_t) + E(e_n) + E(e_r)$$
$$= E(h_t) + 0 + 12.5$$
$$= E(h_t) + 12.5$$

3. If the data are rounded down, then e_r is $U[a,b] = U[-25,0]$

$$E(h_u) = E(h_t) + E(e_n) + E(e_r)$$
$$= E(h_t) + 0 - 12.5$$
$$= E(h_t) - 12.5$$

Since the rounding error e_r depends upon the value of $(h_t + e_n)$, the variance of the UAT geometric height estimate, $\mathrm{Var}(h_u)$, is:

$$\mathrm{Var}\,(h_u) = \mathrm{Var}\,(h_t + e_n + e_r)$$

$$= \mathrm{Var}\,(h_t + e_n) + \mathrm{Var}(e_r) + 2\mathrm{Cov}((h_t + e_n), e_r)$$

$$= \mathrm{Var}(h_t + e_n) + \mathrm{Var}(e_r) + 2[E((h_t + e_n) \cdot e_r) - E(h_t + e_n) \cdot E(e_r)]$$

Let GPS = $(h_t + e_n)$ to represent the GPS system, then $\mathrm{Var}(h_u)$ is

$$= \mathrm{Var}(GPS) + \mathrm{Var}(e_r) + 2[E((GPS) \cdot e_r) - E(GPS) \cdot E(e_r)]$$

1. If e_r is $U[a,b] = U[-12.5,12.5]$, then

$$\mathrm{Var}(h_u) = \mathrm{Var}(GPS) + 625/12 + 2E((GPS) \cdot e_r) - 0$$

2. If e_r is $U[a,b] = U[0,25]$, then

$$\mathrm{Var}(h_u) = \mathrm{Var}(GPS) + 625/12 + 2 \cdot E((GPS) \cdot e_r) - 2 \cdot E(GPS)(12.5)$$

$$= \mathrm{Var}(GPS) + 625/12 + 2 \cdot E((GPS) \cdot e_r) - 25 \cdot E(GPS)$$

3. If e_r is $U[a,b] = U[-25,0]$, then

$$\mathrm{Var}(h_u) = \mathrm{Var}(GPS) + 625/12 + 2 \cdot E((GPS) \cdot e_r) + 2 \cdot E(GPS)(12.5)$$

$$= \mathrm{Var}(GPS) + 625/12 + 2 \cdot E((GPS) \cdot e_r) + 25 \cdot E(GPS)$$

Appendix B
1090 Extended Squitter (ES) Aircraft Geometric Height Modeling

Aircraft geometric height can be obtained from the acquisition of 1090 data when an aircraft is transponding with an extended squitter. There are three models considered for the expected value and variance of the 1090 ES geometric height. The three models are; 1) rounding up or down to the nearest 25 ft increment, 2) rounding up to the nearest 25 ft increment and, 3) rounding down to the nearest 25 ft increment.

If we let d_r = round (GPS height – pressure altitude height), where GPS height is the geometric height from the 1090 ES, then

h_{pa} = round (pressure altitude height)

It follows that the geometric height estimate of the aircraft from the 1090 ES is

$h_{es} = d_r + h_{pa}$

Taking into account the random errors associated with the GPS and pressure altitude measurement systems, we have

$h_{GPS} = h_t + e_n$; where h_t = true geometric height of the aircraft, and
e_n = random error of the GPS measurement system, modeled as $N(0,\sigma)$

$h_{pa} = h_{pat} + e_{pa}$; where h_{pat} = true pressure altitude of the aircraft, and
e_{pa} = random error of the pressure altitude measurement system, modeled as $N(0,\sigma)$

Taking into account the rounding errors in the system we can rewrite d_r and h_{pa} as

$d_r = (h_t + e_n) - (h_{pat} + e_{pa}) + e_{rd}$ where e_{rd} = difference rounding error, modeled as $U[a,b]$

and

$h_{pa} = h_{pat} + e_{pa} + e_{rpa}$ where e_{rpa} = pressure altitude rounding error, modeled as $U[a,b]$

Then, the 1090 ES geometric height of the aircraft can be rewritten as:

$$\begin{aligned} h_{es} &= d_r + h_{pa} \\ &= (h_t + e_n) - (h_{pat} + e_{pa}) + e_{rd} + h_{pat} + e_{pa} + e_{rpa} \\ &= h_t + e_n + e_{rd} + e_{rpa} \end{aligned}$$

1. In the scenario when the rounding is to the nearest 25 ft increment, the rounding error is defined on $U[-12.5,12.5]$. The expected value and variance of the geometric height obtained from the 1090 ES are:

$E(h_{es}) = E(h_t + e_n + e_{rd} + e_{rpa})$

$= E(h_t) + E(e_n) + E(e_{rd}) + E(e_{rpa})$

$= E(h_t) + 0 + 0 + 0$

$= E(h_t)$

$$
\begin{aligned}
Var(h_{es}) &= Var(d_r + h_{pa}) \\
&= Var(h_t + e_n) + Var(e_{rd}) + Var(e_{rpa}) + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) \\
&\quad - 2 \cdot E(h_t + e_n - h_{pat} - e_{pa}) \cdot E(e_{rd}) + 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) - 2 \cdot E(h_{pat} + e_{pa}) \cdot E(e_{rpa}) \\
&= Var(h_t + e_n) + 625/12 + 625/12 + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) - 0 \\
&\quad + 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) - 0 \\
&\approx Var(h_t + e_n) + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) + 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) + 104
\end{aligned}
$$

2. In the case where the 1090 ES data are always rounded up to the nearest 25 ft increment, the rounding error is defined on U[0,25]. The expected value and the variance of h_{es} can be expressed as:

$$
\begin{aligned}
E(h_{es}) &= E(h_t + e_n + e_{rd} + e_{rpa}) \\
&= E(h_t) + E(e_n) + E(e_{rd}) + E(e_{rpa}) \\
&= E(h_t) + 0 + 12.5 + 12.5 \\
&= E(h_t) + 25
\end{aligned}
$$

This gives the true geometric height of the aircraft as;

$E(h_t) = E(h_{es}) - 25$

And variance of of the 1090 ES geometric height, h_{es}, is:

$$
\begin{aligned}
Var(h_{es}) &= Var(d_r + h_{pa}) \\
&= Var(d_r) + Var(h_{pa}) \\
&= Var((h_t + e_n) - (h_{pat} + e_{pa}) + e_{rd}) + Var(h_{pat} + e_{pa} + e_{rpa}) \\
&= Var((h_t + e_n) - (h_{pat} + e_{pa})) + Var(e_{rd}) + 2 \cdot Cov((h_t + e_n) - (h_{pat} + e_{pa}), e_{rd}) + \\
&\quad Var(h_{pat} + e_{pa}) + Var(e_{rpa}) + 2 \cdot Cov(h_{pat} + e_{pa}), e_{rpa}) \\
&= Var(h_t + e_n) - Var(h_{pat} + e_{pa}) + Var(e_{rd}) + 2 \cdot Cov((h_t + e_n) - (h_{pat} + e_{pa}), e_{rd}) + \\
&\quad Var(h_{pat} + e_{pa}) + Var(e_{rpa}) + 2 \cdot Cov(h_{pat} + e_{pa}), e_{rpa}) \\
&= Var(h_t + e_n) + Var(e_{rd}) + Var(e_{rpa}) + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) e_{rd}) - \\
&\quad 2 \cdot E(h_t + e_n - h_{pat} - e_{pa}) \cdot E(e_{rd}) + 2 \cdot E((h_{pat} + e_{pa}) e_{rpa}) - 2 \cdot E(h_{pat} + e_{pa}) \cdot E(e_{rpa}) \\
&= Var(h_t + e_n) + 625/12 + 625/12 + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) \\
&\quad - 25 \cdot [E(h_t + e_n) - E(h_{pat} + e_{pa})] + 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) - 25 \cdot E(h_{pat} + e_{pa}) \\
&\approx Var(h_t + e_n) - 25 \cdot [E(h_t + e_n) - E(h_{pat} + e_{pa})] - 25 \cdot E(h_{pat} + e_{pa}) \\
&\qquad + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) + 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) + 104 \\
&\approx Var(h_t + e_n) - 25 \cdot E(h_t + e_n) + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd})
\end{aligned}
$$

$$+ 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) + 104$$

3. In the case where the 1090 ES data are always rounded down to the nearest 25 ft increment, the rounding error is defined on U[-25,0]. The expected value and the variance of h_{es} can be expressed as:

$$\begin{aligned} E(h_{es}) &= E(h_t + e_n + e_{rd} + e_{rpa}) \\ &= E(h_t) + E(e_n) + E(e_{rd}) + E(e_{rpa}) \\ &= E(h_t) + 0 - 12.5 - 12.5 \\ &= E(h_t) - 25 \end{aligned}$$

$$\begin{aligned} Var(h_{es}) &= Var(d_r + h_{pa}) \\ &= Var(h_t + e_n) + Var(e_{rd}) + Var(e_{rpa}) + 2E((h_t + e_n - h_{pat} - e_{pa})e_{rd}) \\ &\quad - 2 \cdot E(h_t + e_n - h_{pat} - e_{pa}) \cdot E(e_{rd}) + 2 \cdot E((h_{pat} + e_{pa})e_{rpa}) - 2 \cdot E(h_{pat} + e_{pa}) \cdot E(e_{rpa}) \\ &= Var(h_t + e_n) + 625/12 + 625/12 + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) \\ &\quad + 25 \cdot [E(h_t + e_n) - E(h_{pat} + e_{pa})] + 2 \cdot E((h_{pat} + e_{pa}) \cdot e_{rpa}) + 25 \cdot E(h_{pat} + e_{pa}) \\ &\approx Var(h_t + e_n) + 25 \cdot E(h_t + e_n) + 2 \cdot E((h_t + e_n - h_{pat} - e_{pa}) \cdot e_{rd}) \\ &\quad + 2 \cdot E((h_{pat} + e_{pa})e_{rpa}) + 104 \end{aligned}$$